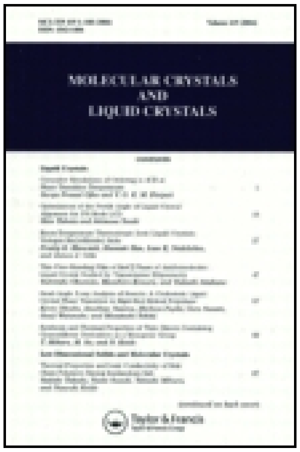

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DESCRIPTION OF DYNAMIC SCATTERING MODE IN LIQUID CRYSTALS BY ANALOGY WITH BLACKBODY RADIATION

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Abstract

A technique for experimental investigation of scattered intensities of LC elements showing dynamic light scattering mode has been developed on the basis of methods of the similarity theory conventionally used for description of blackbody spectral characteristics.

INTRODUCTION

Liquid crystal elements (LCE) based on light scattering effects are extensively used in various optical systems. Two features of the LCEs, namely, non-Lambert indicatrices and controllable shapes of those, do not allow conventional radiometry methods to be applied for energy calculation of optical systems with such LCEs. In Refs. 1-4, mathematical techniques have been developed for energy calculation of optical systems allowing for the non-Lambert indicatrix shapes of the light scattering elements, including LCEs.[5] However, in these papers, the possibility to control light scattering indicatrices was not considered. Noting this possibility, Bertolotti[6] suggested to use LCEs with the dynamic scattering mode (DSM) in nematics as a model radiation source in one of statistic modifications of non-Lmbert radiometry. In the present paper, to describe controllable light scattering in LCEs, we suggest the methods that are commonly used for description of variations in spectral characteristics of blackbody radiation (BBR) with temperature. These methods are conventionally employed in energy calculations of IR instruments.

Since the BBR indicatrix complies with the Lambert law, i. e. it is fixed, the main attention in investigation of BBR is drawn to the spectral characteristics dependence on temperature T which is presented by the Planck radiation law :[7]

$$M_\lambda(\frac{1}{T}) = \frac{1}{\lambda^5} M_1(\frac{1}{T\lambda}) \quad , \qquad (1)$$

where $M_1(\frac{1}{T}) = 2\pi c^2 h \,(\exp \frac{ch}{\kappa T} - 1)^{-1}$,

c is the velocity of light, h is the Planck's constant, κ is the Boltzmann's constant, and M_λ is the radiant emittance at wavelength λ. Functions (1) are plotted in Figure 1,a.

The property of scale similarity in (1) makes it possible to use the following methods of the similarity theory for description of the BBR spectral characteristics:

1. "Kinematic method" . A family of the BBR spectral distribution curves, replotted on logarithmic coordinates, can be represented as successive steps of translational displacement of one "rigid" curve (Figure 1,d).[7]

2. "Catastrophe theory method". A family of the BBR spectral distributions (Figure 1,c) consists of complex curves with distinct maxima that are singular points easily determinable in experiment. The coordinates of such a point (M , λ) as a function of "external action" , that is temperature, is given by the relation:

$$M^0_\lambda(\frac{1}{T}) = \mathcal{B}\, T^5 , \quad \mathcal{B} = 1.2865\ 10^{-11}\ \mathrm{W\ cm^{-3}\ K^{-5}}$$

and $\lambda^0 T = 2897.8\ \mu\mathrm{m\ K}$ – Wien displacement law.[7,8]

3."Dimensionless variables method". With the dimensionless variables

$$\tilde{M} = \frac{M_\lambda}{M^0_\lambda} , \qquad \tilde{\lambda} = \frac{\lambda}{\lambda^0} \quad .$$

all the BBR spectral curves merge into a single "universal curve" (Figure 1,e):[8]

$$\tilde{M} = 142.32\ \tilde{\lambda}^{-5} (\exp \frac{4.9651}{\tilde{\lambda}} - 1).$$

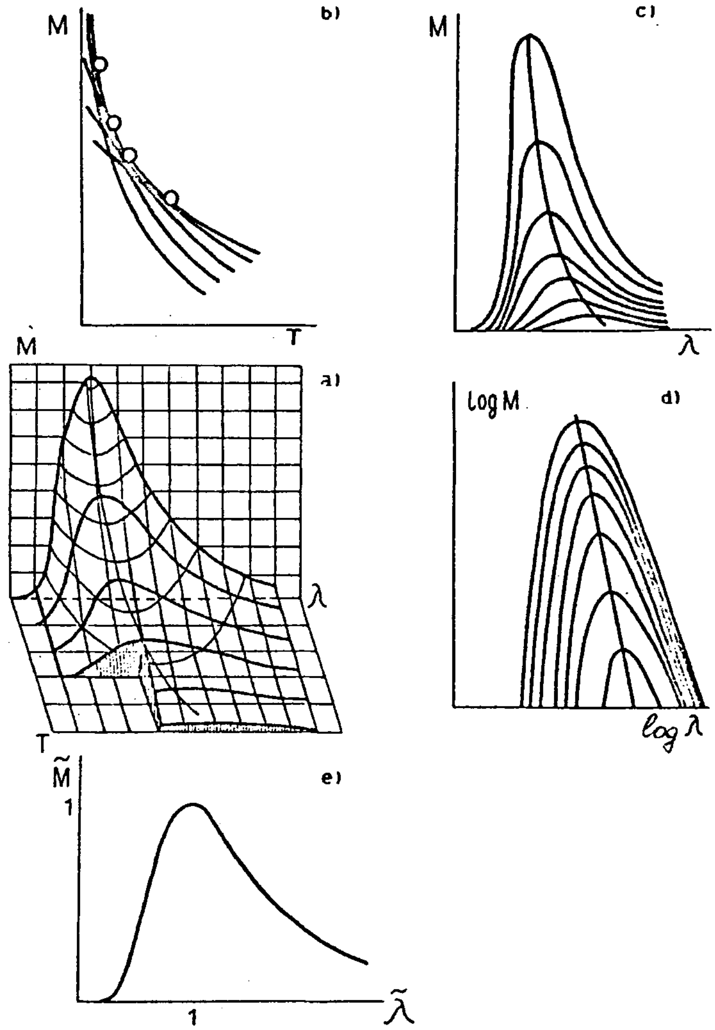

Fig.1. Spectral characteristics of blackbody radiation.

a) Graph of Planck radiation law $M(\lambda, T)$.

b) Family of curves $M_{\lambda}(T)$.

c) Family of isotherms $M_{T}(\lambda)$.

d) Isotherms shift diagrams.

e) Universal curve of blackbody radiation.

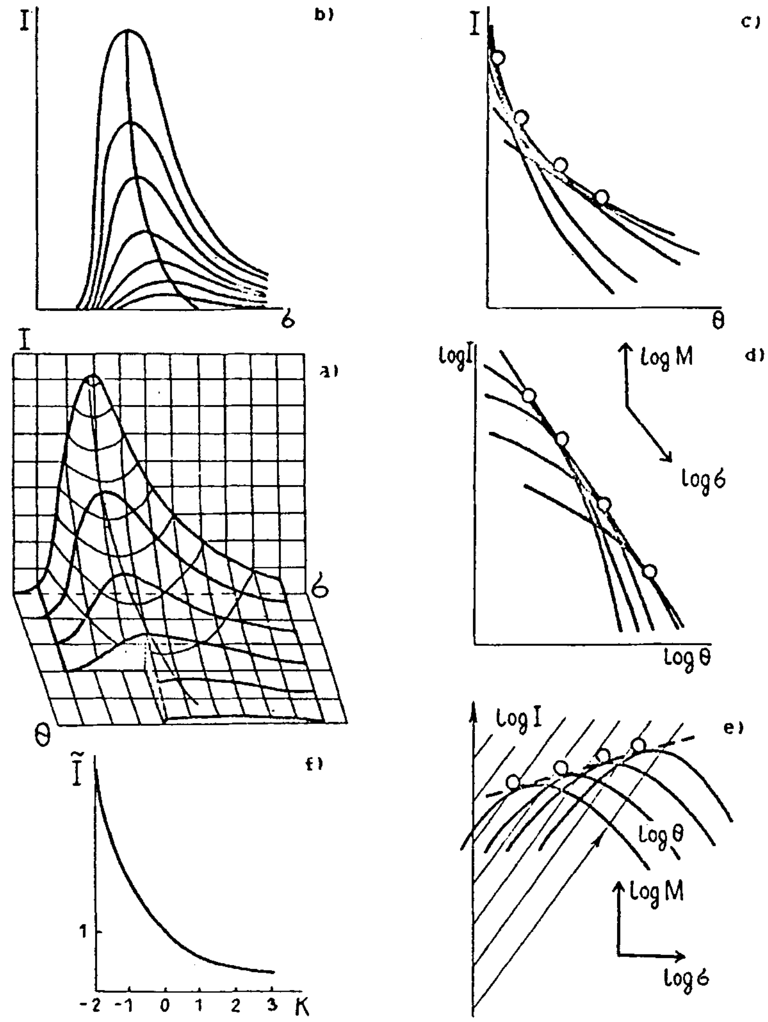

Fig.2. Angular characteristics of LCE radiation with DSM.

a) Intensity I vs scattering angle θ and scale factor σ.

b) Family of curves $I_{\theta}(\sigma)$.

c) Family of indicatrices $I_{\sigma}(\theta)$.

d) Indicatrices shift diagrams in plane of coordinate system with orthogonal axes ($\log\theta$, $\log I$).

e) Indicatrices shift diagrams in plane of coordinate system with orthogonal axes ($\log\sigma$, $\log M$).

f) Universal indicatrices of LCE radiation with DSM.

It is important to note that BBR serves as a model radiation source in conventional Lambert radiometry and its radiometric description is based on the methods of the similarity theory. The controllable light scattering LCE can also be considered as a model radiation source, but for non-Lambert radiometry. The aim of this work is to describe LCE with DSM effect using the techniques developed for description of BBR as a model radiation source in non-Lambert radiometry.

SCALE SIMILARITY IN DYNAMIC SCATTERING MODE

On comparing a family of experimental radiometric curves $\{I_\theta(V)\}$ (Figure 2,b)[9], presenting the scattered intensity I dependence on control voltage V at different scattering angles θ, with a family of isothermal BBR spectral characteristics (Figure 1,c) conforming to the scale similarity condition, one can notice their apparent likeness. To explain the indicatrices similarity, a hypothesis was suggested on the scale change in size of the scattering particle as the control voltage changes.[9,10] This phenomenon was observed in a microscope.[9] A combination of the hypothesis on the scale change in size of the optical inhomogeneities with a notion of LCE as a chaotic phase screen [11] permits theoretical substantiation of the observed similarity in the LCE indicatrices family.

Let us consider the LCE scattering layer as a chaotic phase screen (CPS) [11] that is a thin layer of the optical medium retarding the incident light wave phase by a random value varying with a coordinate. Let CPS be homogeneous and lying in plane q=(x,y). The statistical properties of a homogeneous CPS are described by optical inhomogeneities autocorrelation function $J(\Delta q)$:

$$J(\Delta q) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{U}(q + \frac{\Delta q}{2})\, \mathcal{U}^*(q - \frac{\Delta q}{2})\, dq \ ,$$

where * is the sign of complex conjugation, $\mathcal{U}(q)$ is a complex function, characterizing CPS amplitude transparency distribution.

As function $\mathcal{U}(q)$ modulates complex amplitude $U(q)$ of the incident radiation :

$$U(q) \sim \mathcal{U}(q),$$

therefore statistical properties of the LC layer change the light scattering statistical characteristics and directed radiation obtains statistical properties of the scattering medium, i.e. CPS :[11]

$$\tilde{J}(\Delta q) \sim J(\Delta q),$$

where
$$\tilde{J}(\Delta q) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(q + \frac{\Delta q}{2})\, U^{*}(q - \frac{\Delta q}{2})\, dq$$

is the coherence function.

If LCE is not only homogeneous but is also isotropic , then corresponding coherence function is isotropic too:

$$\tilde{J}(\Delta q) = \tilde{J}(|\Delta q|) .$$

As is generally known[1], the Fourier transform $\mathcal{F}\{...\}$ of the homogeneous and isotropic coherence function is proportional to indicatrix:

$$I(\theta) \approx \mathcal{F}\{\tilde{J}(|\Delta q|)\}.$$

If all optical inhomogeneities in the LCE scattering layer, or CPS, undergo pure scale changes in size by a factor of σ, then, in accordance with the similarity theorem[12], LCE indicatrix is modified into

$$I_{\sigma}(\theta) = \frac{M}{\sigma^2} I(\frac{\theta}{\sigma}) . \qquad (2)$$

Here σ is the optical inhomogeneities scale factor,

M is the proportionality factor (radiant emittance).

Thus, the indicatrix family $\{I_{\sigma(V)}(\theta)\}$ obtained at different voltages can be presented as a result of scale transformation of plane (I, θ) in which the initial indicatrix is plotted (Figure 2,c):

$$(I_{\sigma}, \theta_{\sigma}) = (\frac{M}{\sigma^2} I, \sigma\theta) = (I,\theta)\, (\frac{M}{\sigma^2}, \sigma) .$$

KINEMATIC METHOD FOR DSM INVESTIGATION

It is known that scale transformation in coordinate system ($\log I$, $\log\theta$) is substituted by displacement (Figure 2,d):

$$(\log I_{\sigma},\ \log\theta_{\sigma}) = (\log I,\ \log\theta) + (\log M - 2\log\sigma,\ \log\sigma).$$

Then optical inhomogeneities scale change by a factor of σ in indicatrix family, plotted on logarithmic coordinates, result in the displacement of the initial indicatrix ($\log I$, $\log\theta$) by vector ($-2\log\sigma$, $\log\sigma$) ; i. e. along an axis at angle -arctg2 with respect to abscissa $\log\theta$. If parameter M changes, the initial indicatrix will shift by value $\log M$ along the axis parallel to ordinate $\log I$ (Figure 2,d).

The validity of the scale-similarity condition can be checked by means of a transparency with one of the curves of the family transferred onto it. Then, using it as a reference, a coincidence with all other curves of the family under study is checked by parallel shift of the transparency laid on each curve. The accuracy of the coincidence characterizes the extent of validity of the scale similarity condition for a family of indicatrices measured in experiment. Provided that this condition is satisfied, the description of the transparency motion is sufficient to describe the control voltage effect on parameters mand σ. Since the motion is translational, it is enough to know the motion regularity for any point in the transparency, e.g. some point in the reference curve. However, there is a singular point (I^*, θ^*) tangent to the envelope of the family of curves which is called "attachment point".[13] Its coordinate θ^* is suitable to be defined as the indicatrix halfwidth. It should be noted that scale factor σ is also proportional to the indicatix width $I_{\sigma}(\theta)$, therefore it is convenient to assume value θ^* as scale factor σ :

$$\sigma = \theta^* .$$

The change in light scattering under action of voltage V results in the movement of the attachment point along the envelope. Hence, the regularity $\theta^*(V)$ of the attachment point movement along the envelope describes the voltage effect on scale factor σ. By such definition of parameter σ, the envelope of family of scale similar curves is not only the path of the attachment point but is also function M of σ. The interpretation of the envelope in terms of M and σ is most simple if it is replotted on the coordinates system with orthogonal axis (logM, logσ) (Figure 2,e) but not (logI, logθ) (Figure 2,d). In case of power function $M = e^{\alpha\sigma}$, parameter α can be interpreted as tangent of angle between the envelope and axis logσ (Figure 2,e).

DSM STUDY BY CATASTROPHE THEORY METHOD

The envelope of family of scale similar curves as the path of the attachment point movement is a comprehensive characteristic of scale factor σ and M variation under action of voltage V. From Rene Toma's standpoint,[13] an envelope of family of curves is a singularity of smooth mapping. If a surface

$$\text{graf } I(\sigma,\theta) = \{ \sigma,\theta,I(\sigma,\theta) : \sigma, \theta \in \mathbb{R}_+ \}$$

is constructed on a set of parameters (σ,θ) and by means of that a plane of parameters (I,θ) is mapped in plane (σ,I) (Figure 2,a):

$$\pi: (\sigma,\theta) \longrightarrow (I,\theta) ,$$

then mapping Jacobian degenerates to zero:

$$\det \pi' = 0 \qquad (3)$$

whereat a surface element $d\sigma d\theta$ is mapped in $dId\theta$.

Since this curve is an envelope, the Jacobian degeneration as singularity of smooth mapping is the necessary condition for appearance of the envelope. If the monotonically decreasing curves of the family satisfy the scale-similarity condition (2), equation (3) takes the form:

$$(\log I)'_{\log\theta} = -2 + (\log M)'_{\log\sigma} . \qquad (4)$$

This is another analytic proof that the envelope does not depend on the form of initial function $I_1(\theta)$ of the scale-similar family and also that the envelope is a characteristic of relation between parameters M and σ.

In case when $M = \theta^{\alpha\sigma}$, equation (4) becomes quite simple:

$$(\log I)'_{\log\theta} = -2 + \alpha.$$

It should be noted that the graph of family of functions $\{I_V(\theta)\}$ is concurrently the same of $\{I^*_\theta(V)\}$:

$$I^*_\theta(V) = I_V(\theta).$$

The curves of this family have complex form with distinct maxima, easily checked in experiment. Marking a maximum in one of the curves, we fix three quantities $(I^\circ, \theta^\circ, V^\circ)$ satisfying two conditions:

$$I^\circ = I^*_\theta(V^\circ) \qquad (5)$$

$$I^*_\theta(V^\circ)'_V\Big|_{V=V^\circ} = 0. \qquad (6)$$

It follows from conditions (5) and (6) ,[13] that the maximum is the sought point of attachment to the envelope of family $\{I_V(\theta)\}$. The envelope $I^\circ(\theta^\circ)$ and attachment point motion along the envelope with varying control voltage, $\theta^\circ(V^\circ)$, can be constructed by the maxima of family of curves $\{I^*_{\theta^\circ}(V)\}$.

If curves of family $\{I_V(\theta)\}$ are similar in scale, then the same applies to those of family $\{I^*_\theta(\sigma)\}$:

$$I^*_\theta(\sigma) = \frac{M}{\theta^2} I^*_1(\frac{\sigma}{\theta}) ,$$

where

$$I^*_1(\frac{\sigma}{\theta}) = (\frac{\theta}{\sigma})^2 I_1(\frac{\theta}{\sigma}) .$$

This means that curves of family $\{I^*_\theta(V)\}$ are also similar, provided that control voltage is measured in units of scale factor σ. In case of nonlinear dependence of σ on V, the curves of family $\{I^*_\theta(V)\}$ no longer satisfy the scale similar condition , but they have maxima required for experimental study with the catastrophe theory techniques.

DSM RESEARCH NON-DIMENSIONAL VARIABLES METHOD

The use of dimensionless coordinates:

$$\tilde{I} = \frac{I}{I^\circ} , \quad \tilde{\theta} = \frac{\theta}{\theta^\circ} .$$

is suitable for checking the scale similarity condition in a family of indicatrices. If the indicatrices are similar in scale, then being replotted on dimensionless coordinates $(\tilde{I}, \tilde{\theta})$, all of them will merge into a single "universal indicatrix" $\tilde{I}(\tilde{\theta})$. An accuracy of their coincidence will make it possible to estimate the extent of validity of the scale similarity condition for a family of indicatrices measured in experiment. Unless this condition is satisfied, the family of curves under study on dimensionless coordinate will take the form of fan of rays intercepting point (1,1). In order to simplify the experimental data processing, it is suitable to choose a family of scattering angles $\{\theta_i\}$ as to form geometric progression:

$$\theta_{i+1} = a\, \theta_i ,$$

where a is the progression factor (commonly $\theta_1 = 1^\circ$, a = 2). We assumed that in the attachment point the family of scale factors σ_i corresponds one-to-one to the family of angles $\{\theta_i\}$. Consequently, the family of corresponding voltages V is, in essence, a discrete presentation of regularity of the attachment point motion along the envelope and a description of scale factor σ dependence on V. Each angle θ_i corresponds to the relevant coordinates of the attachment point (σ_i, I_i). The set of points $\{I_i\}$ forming the envelope of family of indicatrices, describes function M of σ.[9] Let I_i^j be the intensity measured at parameters θ_i and σ_i in different combinations. In case of scale similarity of indicatrices of family $\{I_i(\theta)\}$, the "universal indicatrix" :

$$\tilde{I}(\kappa) = \frac{I^j_{j+k}}{I^j_j} ,$$

measured point-by-point, does not depend on j (Figure 2,f). The validity of this condition can be easily checked in experiment since the indicatrices replotted in such a manner, can occur to be not coincident only along axis $\tilde{I}$.[14]

SUMMARY

It has been shown that with scaled change in size of light scattering inhomogeneities in the LC layer, the indicatrices of such LCE also conform to the relations of scale similarity. The methods of similarity theory, conventionally used for description of spectral characteristics of the blackbody radiation, are suggested to describe light scattering in such elements. Particularly, the envelope of family of indicatrices and the regularity of envelope-wise motion of the point of attachment to a specific indicatrix are suggested to be used as a light scattering characteristic measured in experiment.

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